Grood
Math 25

1/24/17

Worksheet 3 - Practice with Integration by Parts

1. Solve the following integrals using integration by parts. (NOTE: You may also need to use substitution in order to solve the integral.)

a) $\int (x+2)e^x \, dx$ b) $\int_1^2 x^3 \ln x \, dx$ c) $\int \arcsin x \, dx$

d) $\int x^3 e^{x^2} \, dx$ e) $\int_0^1 y\sqrt{3y+1} \, dy$ f) $\int \cos\sqrt{t} \, dt$ (Hint: begin with the substitution $w = \sqrt{t}$.)

2. For the following integrals, you will need perform integration by parts more than once to solve it.

a) $\int x^2 \sin(3x) \, dx$ b) $\int e^x \sin x \, dx$ c) $\int \cos^2\theta \, d\theta$ If you remember your double angle formulae from trig, you can solve the last integral above without integration by parts.

3. a) What is $\int (f(x)g(x))' \, dx$? (That is, what is the family of antidervatives of $(f(x)g(x))'$?)

b) Now use the product rule to write $\int (f(x)g(x))' \, dx$ as the sum of two integrals.

c) Use parts (a) and (b) to derive the integration by parts formula.

4. 0 = 1???!! Below, we use integration by parts on $\int \frac{1}{x} \, dx$ to "show" that 0=1. Find the mistake(s) in the argument.

$$\begin{aligned} u = \frac{1}{x} &\Rightarrow du = -\frac{1}{x^2}. \\ dv = dx &\Rightarrow v = x. \\ \text{So } \int \frac{1}{x} \, dx &= \left(\frac{1}{x}\right) x - \int -\frac{1}{x^2} \, dx \\ &= 1 + \int \frac{1}{x} \, dx. \\ \text{So } \int \frac{1}{x} \, dx &= 1 + \int \frac{1}{x} \, dx. \\ \text{Thus, } \quad 0 &= 1. \end{aligned}$$

5. a) Use integration by parts to prove the reduction formula:

$$\int (\ln x)^n \, dx = x(\ln x)^n - n \int (\ln x)^{n-1} \, dx.$$

b) Evaluate $\int (\ln x)^3 \, dx$ using the reduction formula above.

6. SOME REVIEW! Use substitution to solve the following integrals.

a) $\int (x^3+1)^3 x^2 \, dx$ b) $\int \sin^7(2x)\cos(2x) \, dx$ c) $\int \frac{\sqrt{1+\sqrt{x}}}{\sqrt{x}} \, dx$ d) $\int \frac{e^x}{1+e^{2x}} \, dx$

7. Given the following table of values:

x	0	1	$\frac{\pi}{2}$	e	3
$f(x)$	5	7	8	10	11
$f'(x)$	2	4	6	9	12

evaluate the following integrals.

a) $\displaystyle\int_0^1 f'(x)\sin(f(x))\,dx$

b) $\displaystyle\int_1^3 \frac{f'(x)}{f(x)}\,dx$

c) $\displaystyle\int_0^1 e^x f'(e^x)\,dx$

d) $\displaystyle\int_0^1 f'(x)\sin(f(x))\,dx$